

ESTIMATION OF COEFFICIENTS OF THERMAL RESPONSE TEST: THE CHOICE OF THE COORDINATES SPACE OF THE RANDOM FUNCTION

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Extended abstract

In the shallow geothermal sector, the main thermal properties of a reservoir are usually deduced by the well – known production test known as Thermal Response Test (TRT), that is based on thermally stimulating a Borehole Heat Exchanger (BHE) and then recording the fluid average temperature evolution over time, $T(t)$. The aim is the evaluation of equivalent thermal parameters (thermal conductivity, thermal capacity) of the volume of ground interested by the heat exchange, with contemporary verification of BHE thermal properties, as well. In a homogeneous and isotropic medium, the ground volume is a cylinder centered in the BHE, with height equal to the thermo active part of the BHE, and with radius increasing as much as the thermal stimulation lasts. The TRT's common duration varies from a minimum of one day to a maximum of five days, depending on boundary conditions and test's aims and scopes.

Among all several possibilities to deduce equivalent thermal parameters, the simplest and most popular way is the application of the simplified version of the Infinite Linear Source (ILS) solution [1, 2]. This approach assumes the BHE as a borehole of infinite length in a homogeneous and isotropic medium and it expresses the temporal evolution of fluid temperature by (1):

$$T(t) = b \cdot \ln(t) + a \quad (1)$$

The slope b and the intercept a are estimated by operating a classical linear regression on the vector of the experimental fluid data registered at different times. Once identified the slope b , the equivalent underground thermal conductivity λ_g is deduced on the basis of the injected/extracted power rate Q and of the borehole active length, H : $\lambda_g = \frac{Q}{4 \cdot \pi \cdot b \cdot H}$. By the use of the intercept a , a similar procedure allows to deduce the ground volumetric heat capacity c_g and the borehole thermal resistance R_b , linked to one another.

The authors in the last years have introduced a geostatistical analysis of the TRT data starting from the ILS theory [3, 4]. They pointed out the limits of ILS basic assumptions, among which: 1) the vectorial nature of the Regionalized Variable “thermal conductivity”, 2) the meaning of “equivalent thermal conductivity”, 3) the continuous change of support during the test. Nevertheless, the approximated results of TRT given by ILS are useful and currently used to understand and design a shallow geothermal field and the research focuses almost on the quality of parameters' estimations in two approaches (traditional and geostatistical), which are compared by the use of the estimation variance on the regressions of a and b .

The geostatistical model of the TRT considers the fluid temperature registered (Fig. 1) as a Random Function in the domain of time, $T(t)$, whose average is a logarithmic function of time $E[T(t)] = m(t) = a + b \ln(t)$. It is a classical residual model:

$$T(t) = m(t) + Y(t) \quad (2)$$

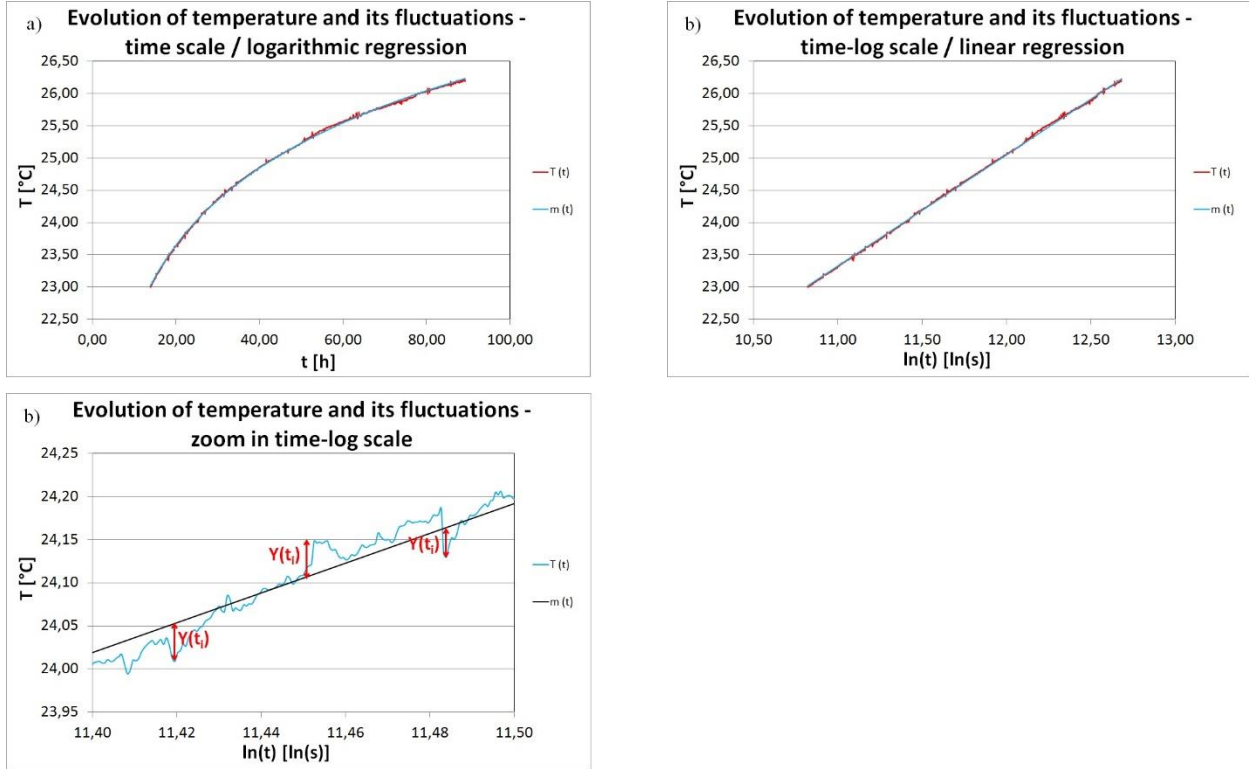


Fig. 1 –Temperature logging during a TRT test in a) the domain of time and in b) the domain of logarithm of time. Zoom in b) the domain of logarithm of time.

A simplified formulation of the formal problem considers only the “least square regression” of the coefficient b that is linear unbiased estimation. The estimation variance is expressed in terms of the variogram of the fluctuations $\gamma(\Delta t) = 0.5 \cdot E \left[(Y(t + \Delta t) - Y(t))^2 \right]$

$$b^* = \sum_{\alpha=1,n} \psi_{\alpha} T(t_{\alpha})$$

$$\sigma_e^2(b^* \rightarrow b) = - \sum_{\alpha=1,n} \sum_{\beta=1,n} \psi_{\alpha} \psi_{\beta} \gamma(t_{\alpha} - t_{\beta}) \quad (3)$$

An analysis of the experimental variogram of residuals, $Y^*(t) = T(t) - m^*(t)$, allows to approximate the stationary variogram model of fluctuations.

The usual regression used in TRT analysis works straightly on the domain of logarithm of time, $\tau = \ln t$, which makes the mean function no more logarithmic, but linear: $m(\tau) = a + b\tau$. Actually the model is $T(\tau) = m(\tau) + Y(\tau)$ and the variogram of fluctuations $\gamma(\Delta \tau) = 0.5 \cdot E \left[(Y(\tau + \Delta \tau) - Y(\tau))^2 \right]$.

The estimated values of the slope b by logarithmic regression in the domain of time and by the linear regression in the domain of logarithm of time do not change, because weights and data keep the same

$$b^* = \sum_{\alpha=1,n} \psi_{\alpha}(t) T(t_{\alpha}) = \sum_{\alpha=1,n} \psi_{\alpha}(\tau) T(\tau_{\alpha})$$

$$\psi_\alpha(t) = \frac{\ln t_\alpha - \overline{\ln t}}{ns_{\ln t}^2} = \frac{\tau_\alpha - \bar{\tau}}{ns_\tau^2} = \psi_\alpha(\tau) \quad (4)$$

Where $\overline{\ln t}$ is the mean of logarithm of times of measures, $T(t_\alpha)$, and $s_{\ln t}^2$ is its variance. The estimation variance in case of stationarity of $Y(\tau)$ has a similar expression.

$$\sigma_e^2(b^* \rightarrow b) = -\sum_{\alpha=1,n} \sum_{\beta=1,n} \psi_\alpha \psi_\beta \gamma(\tau_\alpha - \tau_\beta) \quad (5)$$

Remark that the Random Functions $Y(\tau)$ and $Y(t)$ have the same monovariate distribution in a given point $\tau = \ln t$, so that if one of the two Random Functions is stationary, both are characterized by unique monovariate distribution, with the same mean, zero, and the same variance. But the autocorrelation function is different in the domain of time, $\gamma(t, t + \Delta t)$, and in domain of logarithm of time, $\gamma(\tau, \tau + \Delta \tau)$. Namely if one has a multivariate probability distribution invariant under translation, in general the other has not; that is if one is stationary, in general the other cannot be.

$$\gamma(\Delta \tau) = \gamma(\tau_i, \tau_i + \Delta \tau) = \gamma(\tau_j, \tau_j + \Delta \tau)$$

$$\Delta \tau = \ln(t + \Delta t) - \ln t = \Delta \tau(t, \Delta t) \quad (6)$$

This is due to the non-linear transformation, not of the variable, but of the space of reference of the Random Function. The estimation variance is computed by the same formula and data, but with the variogram of the stationary fluctuation, alternatively $Y(t)$ or $Y(\tau)$.

It is difficult to state a-priori which fluctuation is stationary. Authors suggest to perform the structural analysis in both cases and then to choose the working space where the variogram behaves stationary. In Fig. 2 are shown the experimental variograms of fluctuation in the domain of time and in the domain of logarithm of time for the case study of the experimental TRT presented in Fig. 1. The variogram behaves stationary in the domain of time and a nested structure model, (Nugget + Spherical + Gaussian) has been chosen.

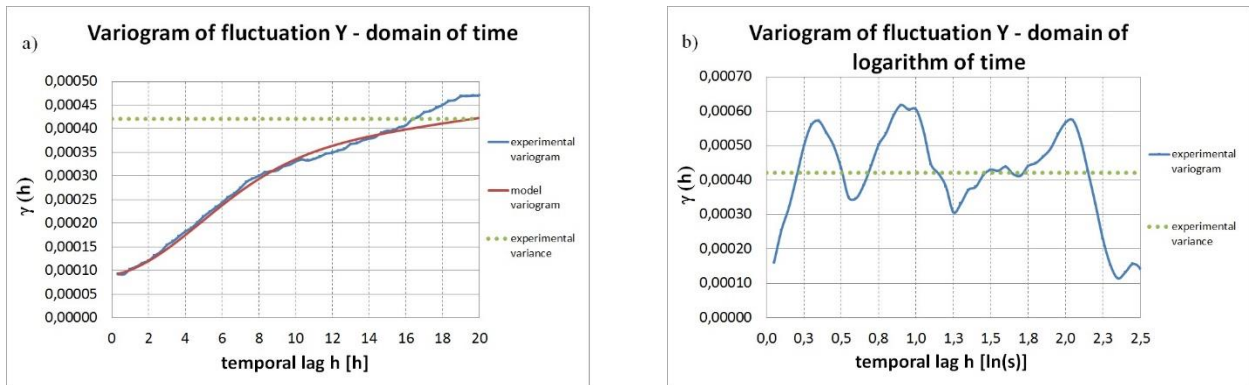


Fig. 2 – Experimental variograms of fluctuation in a) the domain of time and in b) the domain of logarithm of time.

Currently analysis is in progress on theoretical relationships between covariances when applying a non-linear transformation of the domain where a random function is defined. In the particular case study proposed, a coherent modeling of variogram needs to know the relationships between the covariance models $C(t_\alpha, t_\beta)$, $C(\tau_\alpha, \tau_\beta)$ when the transformation $\tau = \ln t$ or $\tau' = e^t$ is applied to the coordinates of a stationary random function $Y(t)$. In addition, the possibility of an experimental inference has to be checked depending on the permanence of the ergodicity hypothesis, given the effect of space transformation on data density.

Keywords

Thermal Response Test, Borehole Heat Exchanger, Equivalent Underground Thermal Conductivity, Estimation Variance, Non-linear Transformation; Stationary Hypothesis.

References

- [1] Eklof C. and Gehlin S. (1996), *TED – A mobile equipment for Thermal Response Tests*, Master's Thesis: 198E, Lulea University of Technology, Sweden.
- [2] Sanner B., Hellström G., Spitler J. and Gehlin S. (2005), *Thermal Response Test – Current Status and World-Wide Application*, Proceedings of World Geothermal Congress 2005, Antalya, Turkey.
- [3] Bruno R., Mercuri S., Tinti F. and Witte H. (2013), *Probabilistic approach to TRT analysis: evaluation of groundwater flow effects and machine - borehole interaction*, Proceedings of European Geothermal Congress 2013 Pisa, Italy.
- [4] Focaccia S., Bruno R. and Tinti F. (2013), *A software tool for geostatistical analysis of thermal response test data: GA-TRT*. Computers and Geosciences 59 (2013), 163 – 170.