Geostatistical Analysis of Geophysical Properties in a Geothermal Reservoir Study

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**Abstract.** Geothermal reservoir models require accurate estimations of rock properties for all nodes/elements of the model domain. However, subsurface data are typically very sparsely distributed. As a consequence thermal conductivity is often considered in a generalized approach for whole geological units.

To better characterize the natural heterogeneity of geological units, an improved approach to predict the spatial distribution of rock thermal conductivity in the subsurface is demonstrated.

Rock thermal conductivity from lab measurements is correlated with bulk density and vertical seismic profile (VSP) velocities from geophysical logging data. Samples are taken from two Rotliegend (Lower Permian) drill cores of approximately 50 m deep boreholes with almost 95 % core recovery. The cores were continuously measured in the lab at a spacing interval of 1 mm or less, summing up to 31,040 and 26,045 data points respectively for dry and water saturated conditions.

The dry data set shows a stronger correlation with density. Generally, the strongest correlation exists between rock thermal conductivity and VSP velocities. The applicability of the secondary variables for kriging (i.e. kriging with an external drift, cokriging and Markov model) was validated using a 1D cross-validation.

For the location where the two drill cores originate from, a 2D p-wave refraction data set is available. By using the observed correlation between seismic velocity and thermal conductivity an improved 2D thermal conductivity interpolation was eventually applied. Ordinary kriging is not able to generate a realistic distribution due to the extreme sparseness of data. Both kriging with a Markov model and kriging with external drift give reliable results, however, some differences exist. The application of co-kriging is limited because of the necessary co-variograms which could not be achieved due to the limited data availability.

**Keywords.** thermal conductivity, petrophysical properties, kriging, secondary variables, geophysical borehole logs, geothermal reservoir modelling

# Introduction

Rock thermal conductivity is governing the most important thermal transport mechanism in the Earth’s lithosphere, but is often not well known in geothermal systems (Naeser & McCulloh, 1989). Geothermal reservoir models require a continuous spatial distribution of rock thermal conductivity values. Typically either thermal conductivity measurements based on core samples in the lab or derived from well logs are used (e.g. Clauser & Huenges, 1995; Fabbri, 2001; Hartmann et al., 2005; Hartmann et al., 2008; Teng & Koike, 2007; Bär et al., 2011; Sepúlveda & Rosenberg, 2012; Vogt et al., 2013, Fuchs & Förster, 2013). In the reservoir model representative average values of each rock formation are assigned as parameters by assuming homogeneous rock properties (e.g. Rühaak et al., 2010). However, even within the same rock type, thermal conductivity can vary over a considerable range (Cermak & Rybach, 1982).

It is well known that bulk thermal conductivity correlates for instance with porosity, density and seismic-wave velocity (Joeleht et al., 2002; Popov et al., 2003; Özkahraman et al., 2004; Hartmann et al., 2005). This correlation results mainly from the fact that the larger the amount of porosity is, the smaller bulk density and thermal conductivity become. This relation is typically mainly linear and was proven independently by numerous studies (Joeleht et al., 2002; Popov et al., 2003; Alishaev et al., 2012; Fuchs & Förster, 2013). Seismic velocity is physically linearly correlated with density. The correlation of density and thermal conductivity is additionally correlated with porosity, due to the inherent physical properties; the molecular structure influences both properties in the same way.

The aim of the study presented here is to derive optimal interpolation approaches where the sparseness of thermal conductivity measurements is overcome by a combined (joined) interpolation using density or p-wave velocities. The latter ones are ideally known from geophysical surveys for large 2D sections or even 3D volumes.

Throughout the study, a rock thermal conductivity interpolation is performed based on drill core samples, borehole geophysical data and 2D seismic profiles. For the computation, GSLIB (Deutsch & Journel, 1992) is used. Cross validation shows reliable estimation results among four kinds of kriging methods: ordinary kriging (OK), kriging with an external drift (KED), colocated cokriging (Markov model) and ordinary cokriging. The latter three methods are particularly used for co-regionalized variables, such as dry bulk rock thermal conductivity correlated with bulk density or seismic velocity. Finally a 2D interpolation is performed.

# Study area

The UNESCO world heritage site Messel pit (Germany) is located on the Sprendlinger Horst, a Paleozoic Horst structure separating the northern Upper Rhine Graben in the West from the Gersprenz Graben in the East (Figure 1). Above crystalline bedrocks of the Variscan basement Permo-Carboniferous volcanics and sediments of the Rotliegend and Middle Eocene sediments have been deposited.

In 2004, the Leibniz Institute for Applied Geophysics (LIAG), Hannover/Germany drilled two boreholes (GA1 and GA2). Complete cores were collected for depths of 68 m and 80 m respectively. Additionally, geophysical borehole logging was conducted to study the Messel Fault Zone. The p-wave velocity of vertical seismic profile (VSP) and bulk density data from density logging are used in the study. Moreover several p-wave 2D seismic sections were recorded; one of them strikes through both boreholes.

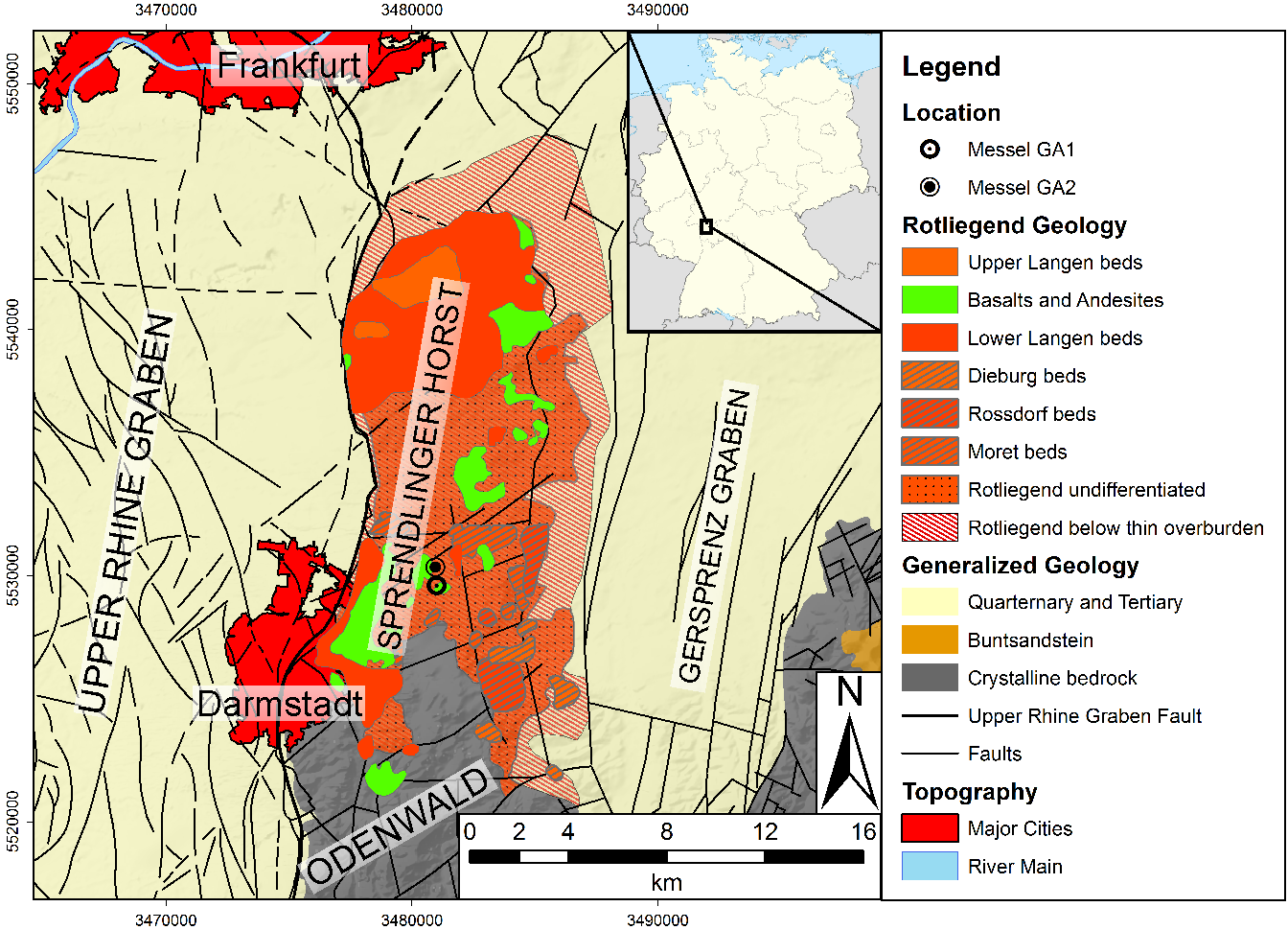


Figure 1: Generalized geological map of the study area on the Sprendlinger Horst, a NE segment of the Upper Rhine Graben shoulder.

# Quasi continuous measurement of thermal conductivities

Cores from GA1 and GA2 boreholes are stored in the archive of the Forschungsinstitut Senckenberg, Grube Messel. The nearly 95 % core recovery allows for the measurement of a continuous rock thermal conductivity profile. Thermal conductivity from the Rotliegend from GA1 borehole at the depth interval of 6 m – 13 m and GA2 borehole at the depth interval of 2.5 m – 50 m was continuously measured in the laboratory. The Rotliegend formation consists of fine-grained sandstones alternating with coarse and fine-grained breccias and conglomerates and well-bedded siltstones.

It is important to keep the measurement facing the down-hole to maintain the same stratigraphic sequence as density logging, VSP velocity and refraction velocity. Values for each sample were measured at a spacing of 1 mm or less along the scanning line, resulting in 31,040 and 26,045 values for dry and water saturated conditions respectively. Dry properties show a broader distribution of values. The representative values of the statistical analysis indicated that although both data sets sample the same stratigraphy, the median and range of the water saturated thermal conductivity are significantly higher than those of dry conditions. The reason is the thermal conductivity of water (0.65 W m-1 K-1) which is substantially larger than that of air (at 20 °C 0.025 W m-1 K-1).

In the following, cross-correlation plots are presented and discussed for different data pairs.

## Rock thermal conductivity and density correlation

To match thermal conductivity and density measurements at the same location, the density values were interpolated linearly to the location of the thermal conductivity measurement. A regression analysis of dry and water saturated thermal conductivity versus in situ density was performed and is shown in Figure 1. Water saturated samples have a broader range than the dry samples. Nevertheless, dry samples have a stronger positive linear relationship between density and thermal conductivity. While regression analysis shows that there is no correlation for the water saturated data (R2 = 8⋅10-4), the correlation for the dry data set is relatively small but considerably larger with R² = 0.156, which indicates that a stronger relationship exists for dry samples because of the larger contrast between rock and pore volume properties for dry samples.

Figure *2*: Correlation of thermal conductivity with bulk density for dry and water saturated conditions. The red line shows the correlation for dry samples and the black line for water saturated samples respectively.

## Rock thermal conductivity and seismic velocity correlation

Both refraction seismic and VSP profiling give compressional p-wave velocities. Rock matrix density is directly correlated with rock matrix thermal conductivity for sedimentary rocks (Sundberg, 2009). Furthermore, bulk density ρ (kg·m-3) is correlated with seismic velocity vp (m·s-1). The latter one depends on the elastic properties as well as bulk densities of the media and varies with mineral content, lithology, porosity, pore fluid saturation and degree of compaction (Wonik, 2007).

Figure 3: Correlation between rock thermal conductivity and VSP velocity. The blue diamond indicates the water saturated conditions while red ones indicate dry conditions.

For the correlation shown in Figure 3, the rock thermal conductivity was interpolated linearly on the position of the VSP seismic measurements. The dry samples still demonstrate a stronger relationship (R2 = 0.53) than the water saturated ones (R2 = 0.18). The correlation of rock thermal conductivity with density is less than that with p-wave velocity. A likely explanation is that bulk densities of natural rocks do not significantly differ (2500 m·s-1 – 2850 m·s-1). The minerals constituting the solid part of sandstones have similar specific gravity values (Özkahramann, 2004).

The seismic refraction velocity from the seismic profiles has a spacing of 15 m in horizontal direction and of 10 m in vertical direction. A direct correlation with the VSP or the thermal conductivity is not possible. However, due to the physics of VSP and refraction data, they are considered to be correlated. This correlation between VSP and thermal conductivity was proved statistically by a t-test.

# Spatial variability

Experimental variograms are a convenient tool for the analysis of spatial data as they are based on a simple measure of dissimilarity. Using the continuous measured dry thermal conductivity at the drill core of GA2 (Figure 4(a)), a vertical variogram was calculated.

The Rotliegend sediments are anisotropic layers; therefore, the horizontal spatial distribution has to be considered. Thermal conductivity data are only available for the vertical direction of the boreholes GA1 and GA2. However, measured thermal conductivities of the Saar Nahe and Wetterau Basins, to which the Sprendlinger Horst belongs as well, are available (Aretz et al., 2013). The data could be used for calculating a horizontal variogram. The samples were collected from more than 100 outcrops with distances between 2 km and 150 km from the Sprendlinger Horst. The laboratory measurement of rock thermal conductivity was conducted similarly as for the GA cores. This time a mean rock thermal conductivity was calculated for each sample. All samples’ mean were used to calculate an omni-directional experimental variogram and to for fitting variogram models (Figure 4(b)).

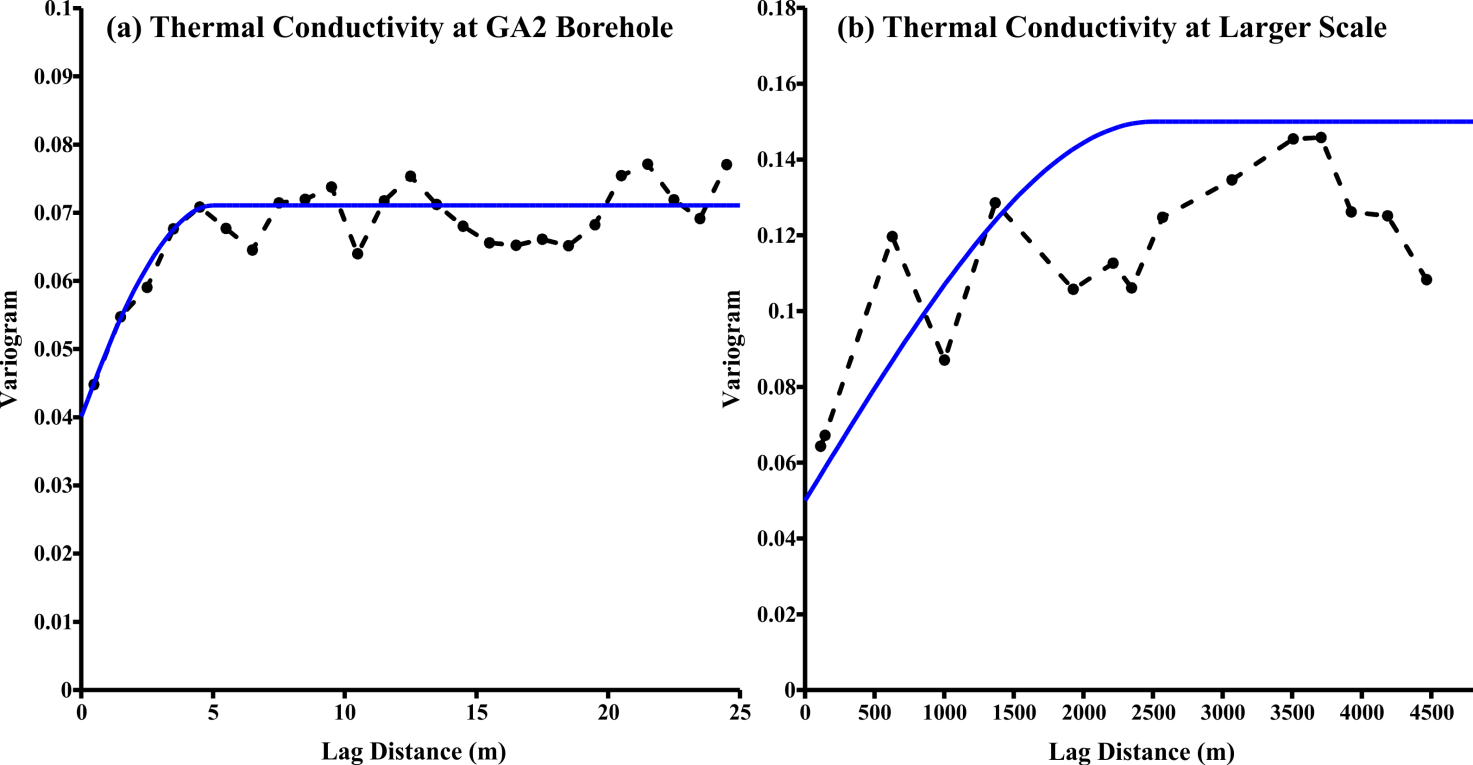


Figure 4: Experimental variograms of thermal conductivity at (a) GA2 borehole and (b) a larger scale in Rotliegend sandstones from the Saar Nahe and Wetterau Basin, Germany. The black points are plotted by experimental data. The blue lines represent variogram models.

The experimental variograms have been fitted by spherical models (Table 1).

Table 1: Parameters of spherical model corresponding to Figure 4

|  |  |  |  |
| --- | --- | --- | --- |
| Data Distribution | Nugget Effect  (W·m-1·K-1)2 | Lag Distance  (m) | Sill  (W·m-1·K-1)2 |
| GA2 | 4.0⋅10-2 | 5 | 3.2⋅10-2 |
| Large scale | 5.0⋅10-2 | 2500 | 1.0⋅10-1 |

# Results

Kriging allows tackling the problem of estimating attribute values at un-sampled locations by a least-squares linear regression algorithm. It accounts for data related solely to the continuous attribute being estimated. Four kriging methods, ordinary kriging (OK), kriging with an external drift (KED), colocated cokriging (Markov model) and ordinary cokriging are applied here. Details for the selected geostatistical approaches can be found in Deutsch & Journel (1992), Hudson & Wackernagel (1994), Kitanidis (1997) and Goovaerts (1997).

## 1D interpolation

For testing the suitability of the different physical properties as secondary variables a cross-validation was performed. The observations at every 20th, 50th, 100th and 500th value were conducted as the input data but with the maintenance of the drift from each of the secondary variable on the same grid. This way only a portion of raw data of 5 %, 2 %, 1 % and 0.5 % is used in the interpolation shown in Figure 5 as well as the plots of 1% of input data estimated by the kriging methods are presented here as an example. The Root Mean Square Error (RMSE) values show the results of cross validation in Figure 6.

Because the neighborhood value strongly influences the linear regression estimator part in the random variable (RV) model by the OK method, the estimated results are concentrated in a mean value from the surroundings. The results are similar to the result of the Markov model, except there are less extreme values and the plotting curve is smooth and maximally continuous. In comparison, the correlation between rock thermal conductivity and other physical properties in the KED method is generally based on their linear relationship. It takes exhaustive information from the secondary variable when the rock thermal conductivity value is missing. Hence, the stronger relationship will obtain the better results.

Although the results of the CK show slight improvement compared to the others through the RMSE comparisons (Figure 6), it need measure a cross spatial dependence between two variables. Otherwise, it does not perform properly.

In addition, it becomes visible that the VSP data as the secondary variable is the best auxiliary variable (Figure 6), superior to density and seismic velocity results. However, in our case VSP data is only available from 20 m downwards.

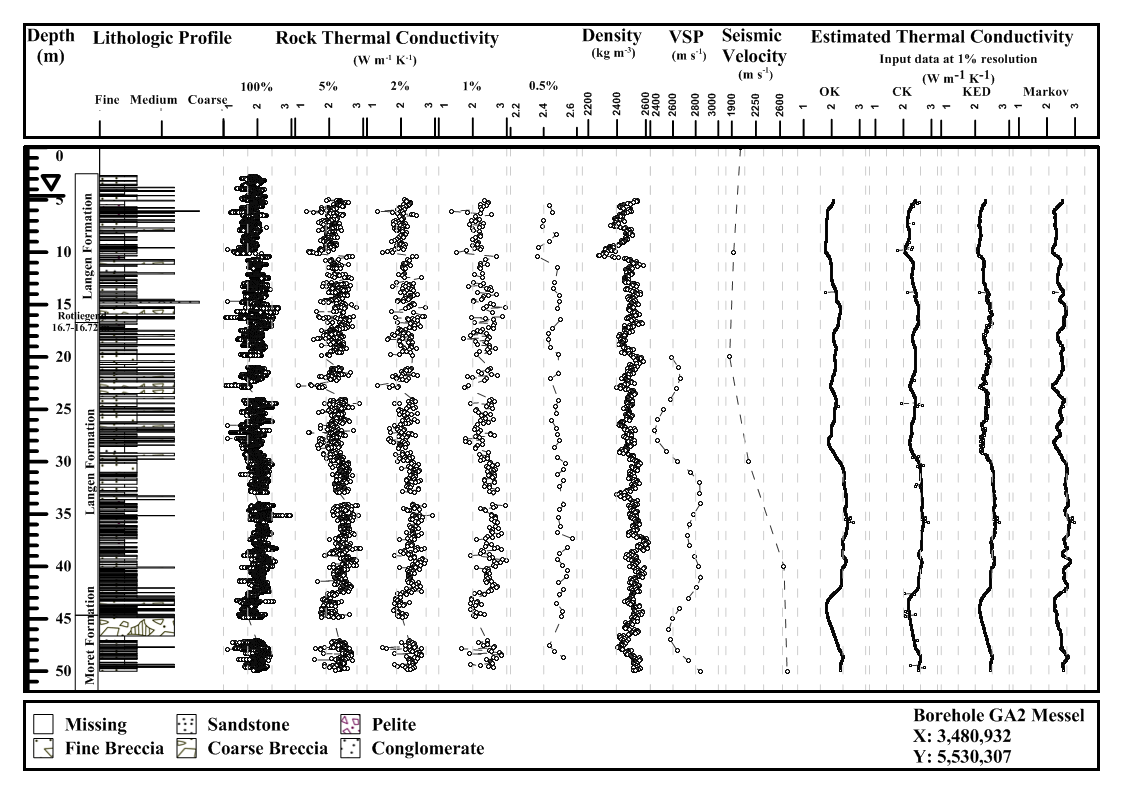


Figure 5: GA2 borehole lithologic profile, laboratory measured dry rock thermal conductivity with five resolutions (100%, 5%, 2%, 1 % and 0.5 %), borehole logging data (density and VSP), seismic refraction data from 2D seismic profile (seismic velocity) and estimated thermal conductivity results when the input data are at 1 % of raw data.

Figure 6: RMSE results of four kriging methods when the input data are at different resolution.

## 2D interpolation

For demonstrating the concept of using seismic data as secondary variable for thermal conductivity interpolations, such interpolation is performed for the measured thermal conductivity together with a 2D seismic profile. A main problem for a 2D interpolation of thermal conductivity is the limited spatial distribution of available measurements, only along the two boreholes. No values in between are known. The area is only continuously covered by refraction seismic velocity. Due to this reason, it was not possible to get a cross-variogram which is why the application of ordinary cokriging could not be utilized here.

The different results are shown in Figure 7. The KED and Markov models are improved by the secondary variable. The variation of KED and Markov model is due to their specific drift between the primary and secondary variables. Rock thermal conductivity is linked by the coefficient of the seismic velocity. This way the estimated data is available at any location where the secondary variable exists. Hence, the correlation coefficient dominates the accuracy of the result. But the rock thermal conductivity in KED method is constrained to local influence by the secondary variable. The OK result is also presented against Markov model and KED here to show the insufficient interpolation without the secondary variable.

It has to be pointed out that in the seismic survey, elastic waves are generated by a vibrator source on the surface. Refraction seismic requires an increase in velocity with depth. If a layer of lower seismic velocity underlies a layer of higher velocity, no refraction at the critical angle occurs and the layer cannot be detected (hidden layer). Hence, such conditions may cause ambiguity in the interpretation. The results of the VSP analysis generally support the seismic refraction profile investigation. But the boundary between Rotliegend and the crystalline basement is not significantly distinct due to the bedrock weathering. It could influence the accuracy of the interpolation results.

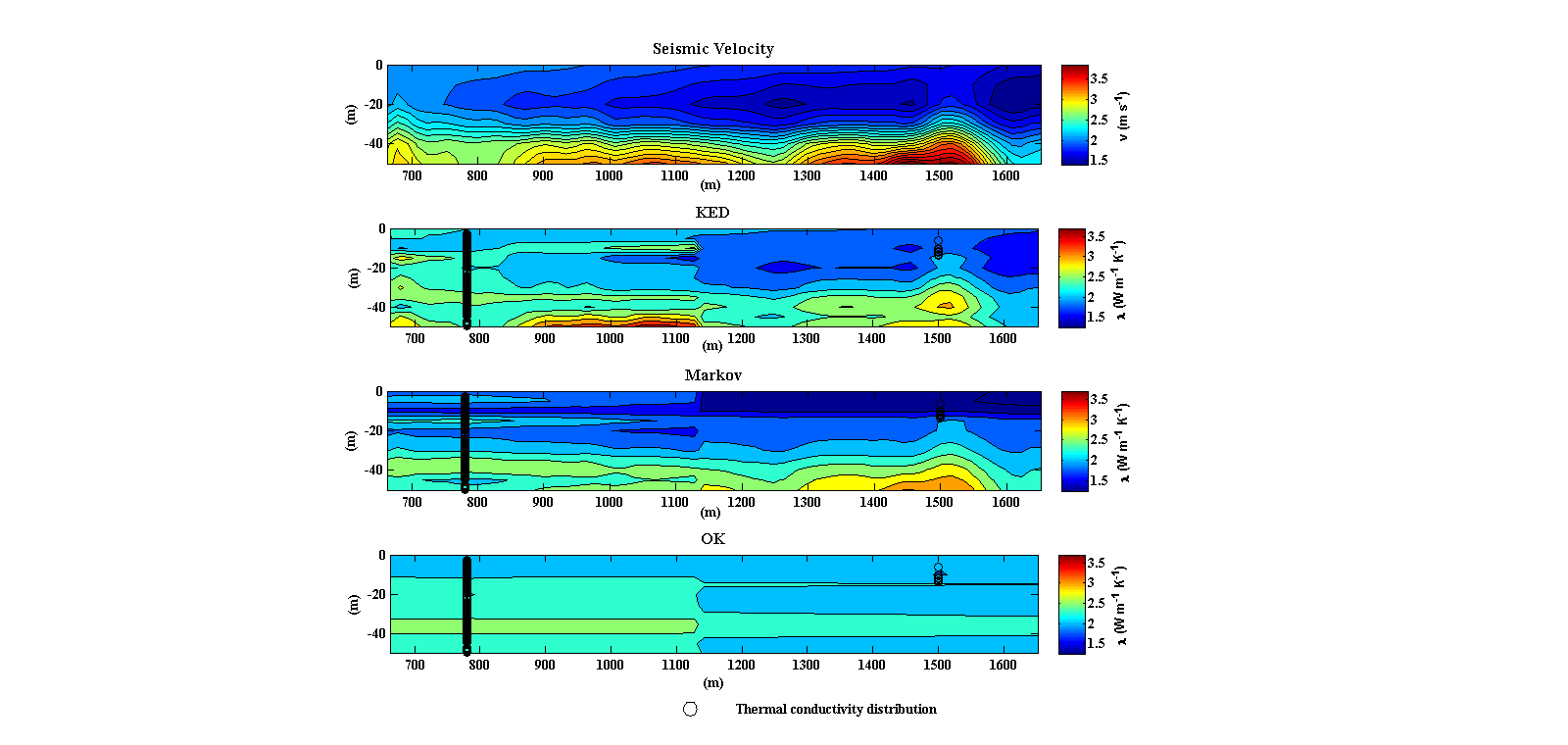


Figure 7: Plots of the 2D refraction seismic profile (top) used to calculate the 2D kriging results estimated by Markov model, KED and OK (from top to bottom).

# Summary and discussion

Based on a large dataset of different kinds of measurements, rock thermal conductivity from the lab, measurements of a set of geophysical borehole logs, an improved interpolation method of thermal conductivity for geothermal reservoir studies is presented.

To overcome the difficulty of data sparseness in variogram analysis, a dense set of rock thermal conductivity measurements was conducted at the spatial interval of 1 mm in the lab. The samples are sandstone of Rotliegend age.

The rock thermal conductivity under dry condition shows a wider range of  W m-1 K-1 than that of water saturated conditions with  W m-1 K-1.

A dense set of measurements is composed to a continuous rock thermal conductivity profile along a borehole and correlated with geophysical well logs. Due to the porosity variation, the dry thermal conductivity profile has a stronger linear regression relationship with bulk density and compressional p-wave velocity. However, the correlation of thermal conductivity with density is not as good as with the p-wave velocity.

Both 1D and 2D interpolation results benefit of a secondary variable which is easier to obtain than continuous thermal conductivity profiles, because geophysical data are usually more readily available. Direct measurements of the spatial distribution of rock thermal conductivity are associated with difficulties and high costs (Sundberg et al., 2009).

With the demonstrated correlation between rock thermal conductivity and density, VSP or seismic velocity, the RMSE results show that the linear model fits reasonably well over the borehole profile. Furthermore, the rock thermal conductivity contour map in 2D interpolation fluctuates due to the influence of the refraction seismic velocity.

There is more information of underground conditions like rock fracturing, weathering and heterogeneities in rock lithology encompassed in the seismic velocity. Usage of a correlated secondary variable is a promising method for efficient and economical interpolation of rock thermal conductivities. However, analysis and interpretation of rock thermal conductivity in the 2D section based on the measurement of two cores alone is challenging. The main difference is how the secondary data is handled. It directly influences the Markov model estimate, which is a relatively global linear correlation between primary and secondary variables as captured by the coefficient. In KED the secondary variable is only used for the locally primary trend; especially, when the estimated slope of the local trend is large, it tends to strongly influence the KED estimate rather than the Markov model.

In general, the result confirms that the support of the secondary variable is a useful way to obtain rock thermal conductivity interpolation in geothermal reservoirs.

Future work will focus on the distribution of the matrix rock thermal conductivity and heat capacity estimation, or include transfer this approach on additional test fields and moreover, to 3D.

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